

# Physics in Games

Matthias Müller

[www.MatthiasMueller.info](http://www.MatthiasMueller.info)



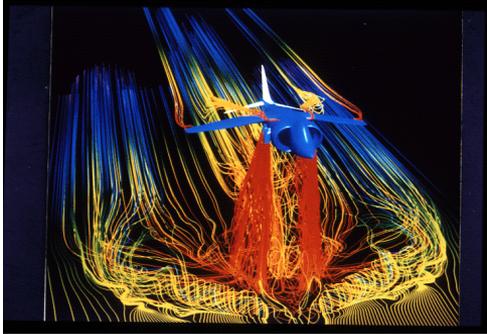
**NVIDIA.**



# Outline

- Comparison
  - Physical simulations in engineering
  - Offline physics in graphics (mostly movies)
  - Interactive physics
  - Real time physics in games
- Position Based Dynamics
  - Algorithm
  - Examples: cloth, rigid bodies, fluids, unified solver
- Q & A

# Simulations in Engineering

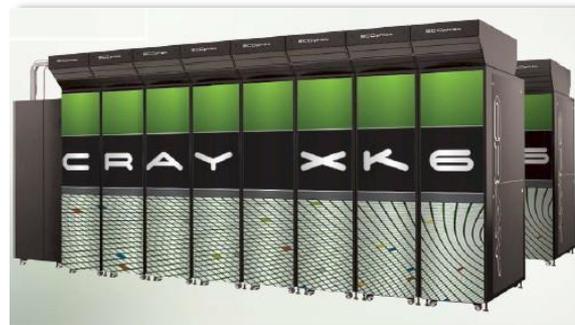
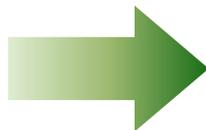


- Complement real experiments
- Extreme conditions, spatial scale, time scale
- Accuracy most important factor
- Low accuracy: Useless result!
- One central gigantic computer

# Evolution of Compute Power

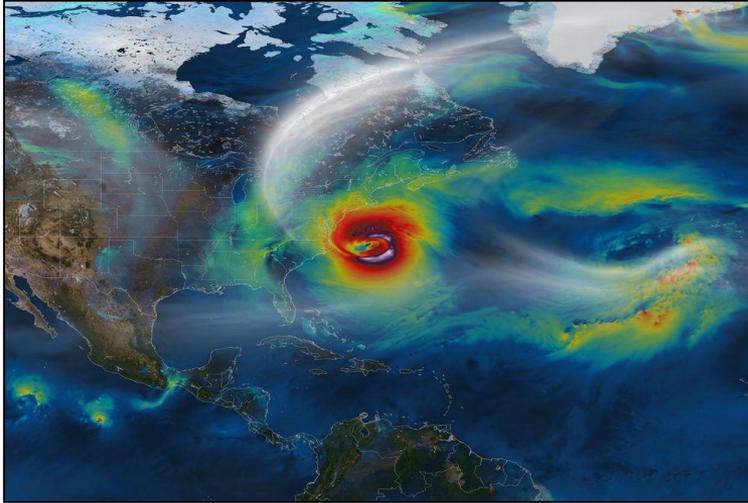


Zuse's Z1 (1938)  
0.2 ops



Titan (currently number 2)  
using 18,000 **nvidia** GPUs  
~27,000,000,000,000,000 flops!

# Simulation of Hurricane Sandy



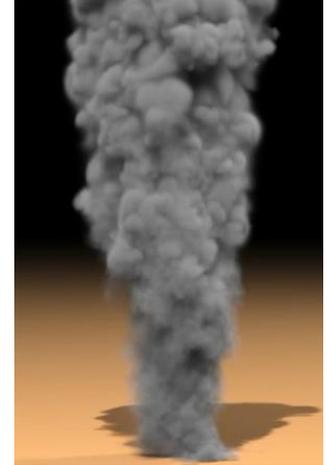
- National Center for Supercomputing Applications
- 9120 x 9216 x 48 cells (500 m)
- 13,680 nodes and 437,760 cores on Titan
- Sustained rate of 285 teraflops

# Physics in Graphics



# Re-inventing the Wheel?

- Since late 80's [Terzopoulos et al. 87, 88]
- Rediscoveries
  - Semi-Lagrangian advection, co-rotational FEM,  
X introduced Y to graphics (SPH, MPM, FLIP, ...)
- Goals of physics in graphics
  - Imitation of physical phenomena / effects
  - Plausible behavior (cheating possible)
  - Trade accuracy for speed, stability, simplicity
  - Control (by director / game developer)
- New goals require new methods!



# Offline Methods



[Emmerich, Movie 2012]

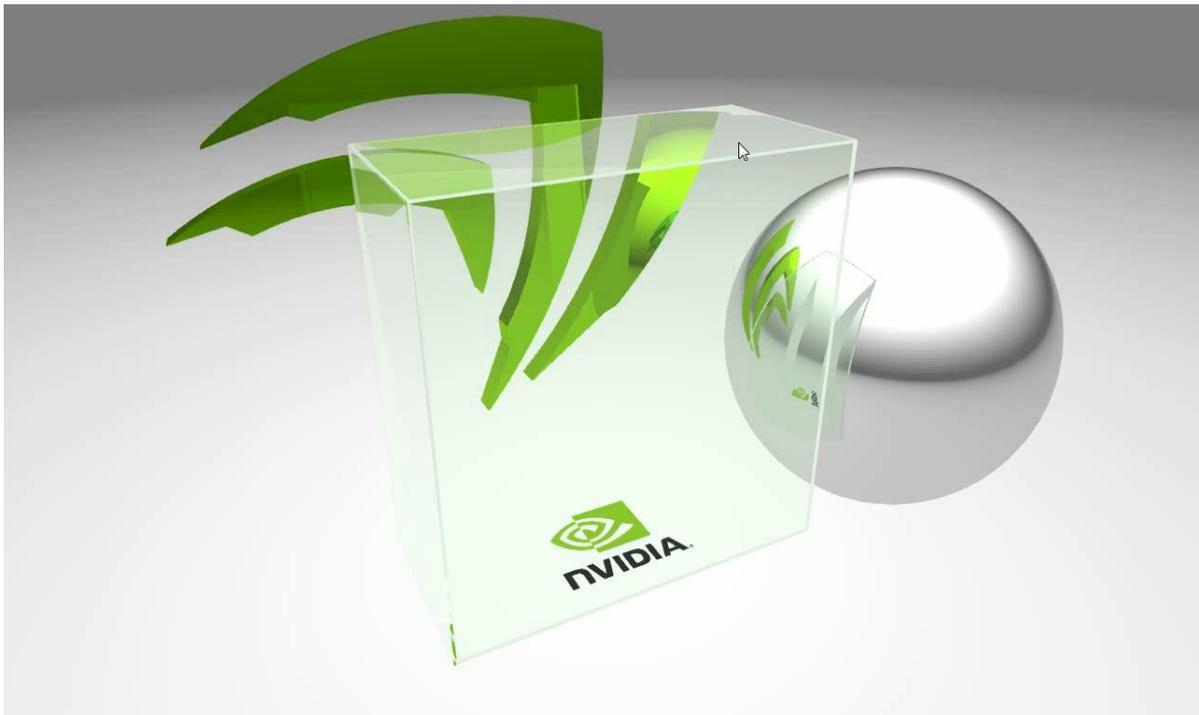
- Main application: **Movies**
- $\gg$  1 sec of computation for 1 sec of simulation allows:
  - **High resolution** (fluid grid, FEM mesh, time steps)
  - **Re-runs** and **adaptive time steps**
  - Time consuming shading

# Interactive Physics

- Between offline and game physics
- Virtual surgery, virtual reality, demos
- All available compute power
- > 15 fps
- No adaptive time steps
- Robust
  - No re-runs
  - Unforeseeable situations

# Water Demo (GTC 2012)

- First time real-time Eulerian water sim + ray tracing



2 x GTX 680

Multi-grid

[Chentanez et al., 2011]

OptiX



# Dragon

- Eulerian fluid simulation + combustion model + volumetric rendering



# Physics in Games



# Game Requirements

- **Cheap** to compute
  - 30-60 fps of which physics only gets a small fraction
- Low **memory** consumption
  - Consoles, fit into graphics (local) memory
- **Stable** in extreme settings
  - 180 degree turns in one time step
- High level of **control**
- **Challenge**
  - Meet all these constraints
  - Get to offline results as close as possible

# Speedup Tricks

- Reduce simulation resolution
  - Simple: Use same algorithms
  - Interesting details disappear
- Reduce dimension (e.g. 3d → 2d)
- Use different resolution for physics and appearance
- Simulate only in active regions (sleeping)
- Camera dependent level of detail (LOD)
- Invent new simulation methods!
- Use nvidia GPUs and CUDA! 😊



# Game Physics Methods



# Animation

- Pros:
  - Can be and **still is** used **for almost everything** (3d movie playback)
  - **Full control**
  - What artists are used to do
- Cons
  - Time consuming manual work
  - Hard to handle complex phenomena
  - Repeating behavior

# Particle Physics

- **Simplest** and **very popular** form of physics effect
  - droplets, smoke, fire, debris [Reeves, 1983]
- **Effects physics** vs. game play physics
  - does **not influence game play**, no path blocking
- Most expensive part:
  - **collision detection** with large environments
  - particle-particle **interaction** (often not needed)
  - Advection by incompressible velocity field (**fluid solver**)



PhysX<sup>™</sup>  
by NVIDIA

# Rigid Bodies

- Game physics engines = rigid body engines
- Challenges
  - **Stability** (stacking)
  - **Speed** (solver and collision detection)
  - **Continuous collision detection** (fast moving objects)
- Rarely in-house
- **Middleware** popular (*PhysX, havok, bullet*)

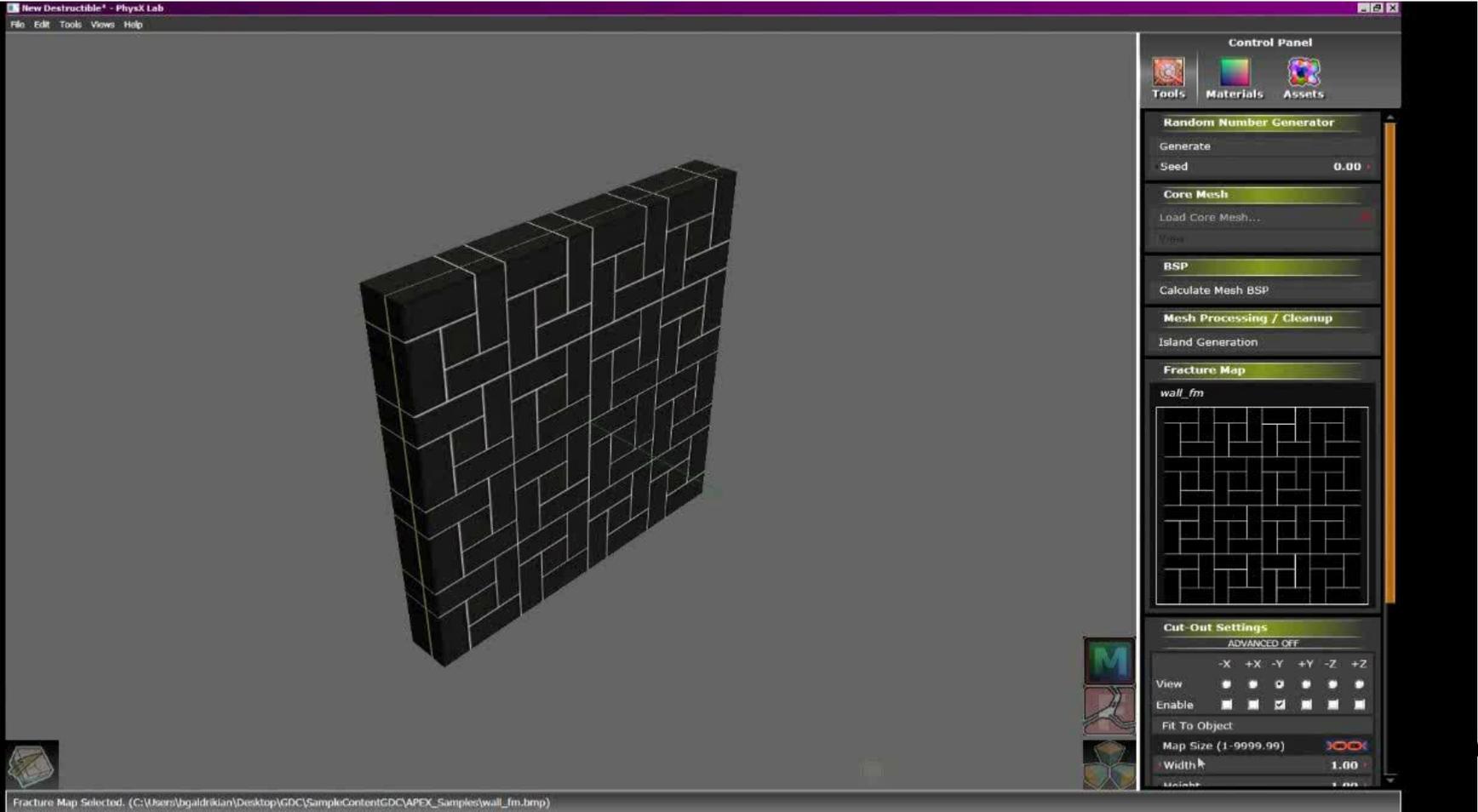


*inthevif* with blender & bullet

# Destruction

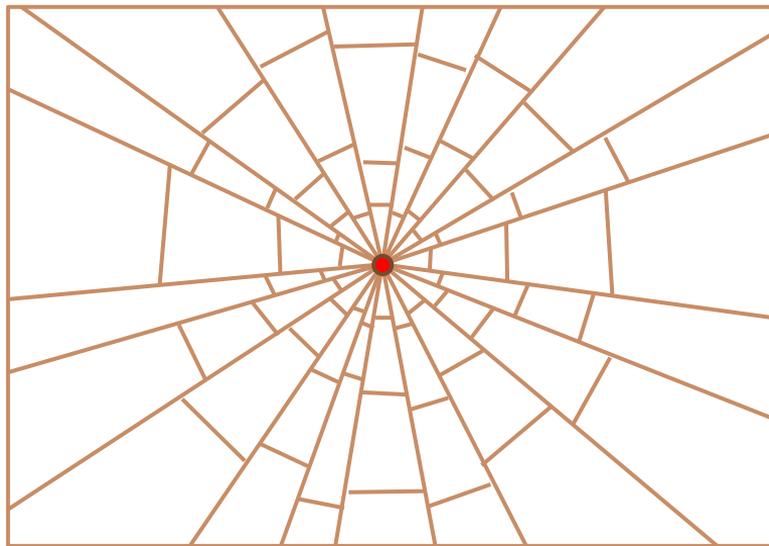
- Traditional: static fracture
- Artists **pre-fracture** models
- Models are replaced by parts when collision forces exceed a threshold
- Pro:
  - High level of control
- Cons:
  - Tedious manual work
  - **Independent of impact location**

# PhysX Destruction Tool



# Pattern Based Fracture

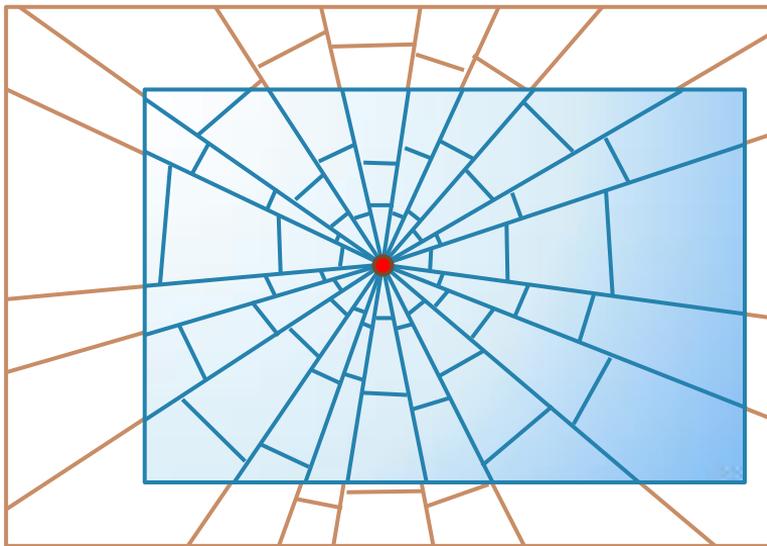
[Müller et al., 2013]



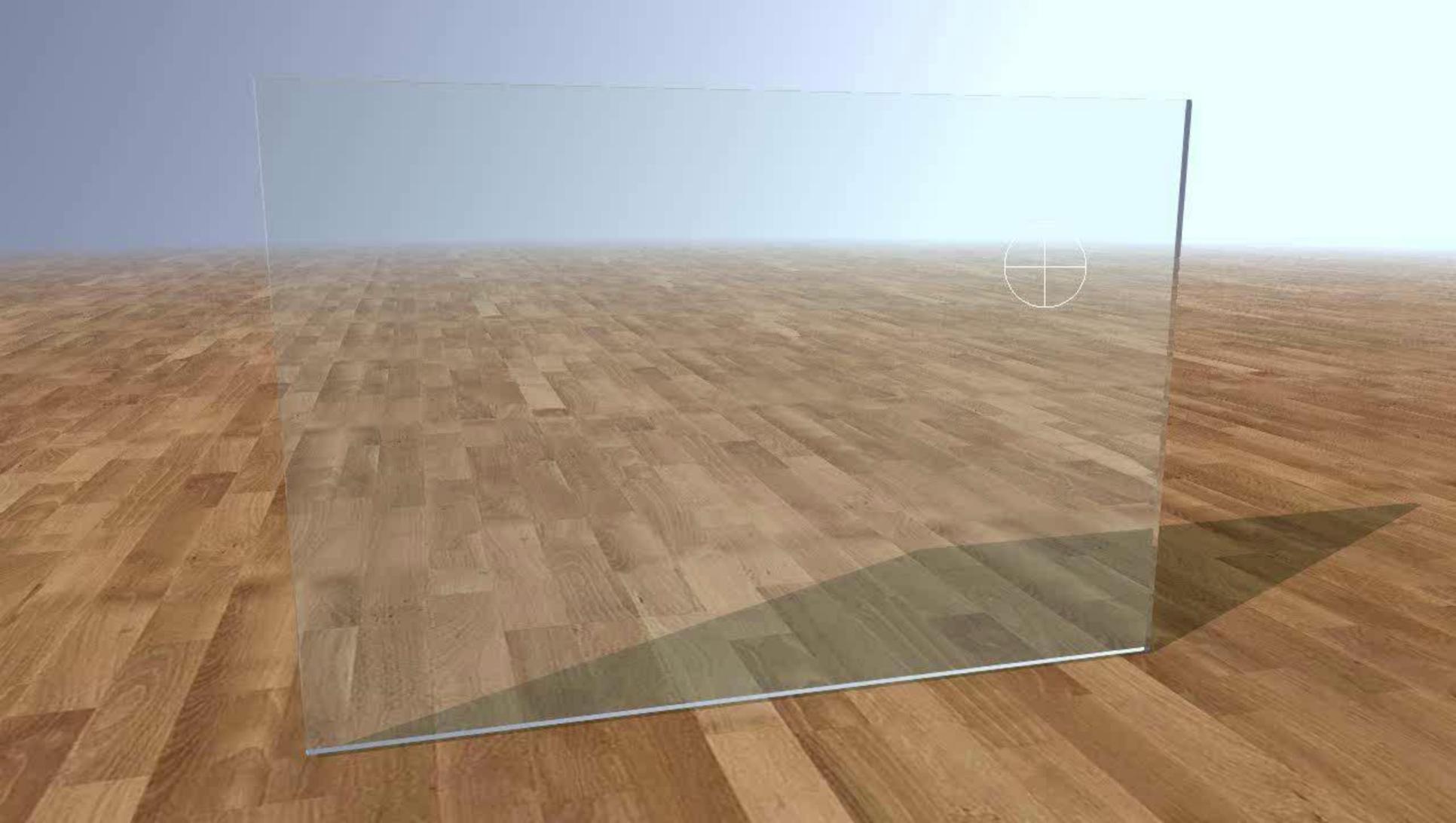
- Pre-designed fracture pattern

# Pattern Based Fracture

[Müller et al., 2013]



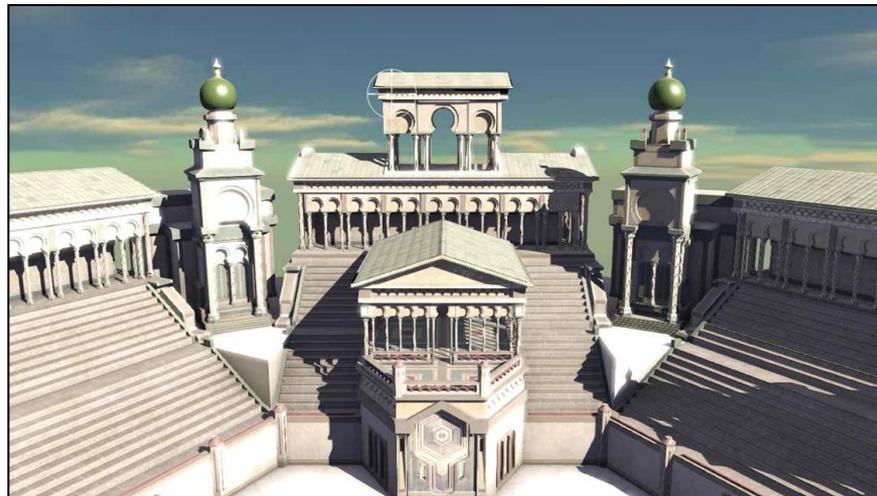
- Pre-designed fracture pattern
- Align pattern with impact location at runtime
- Use pattern as stencil

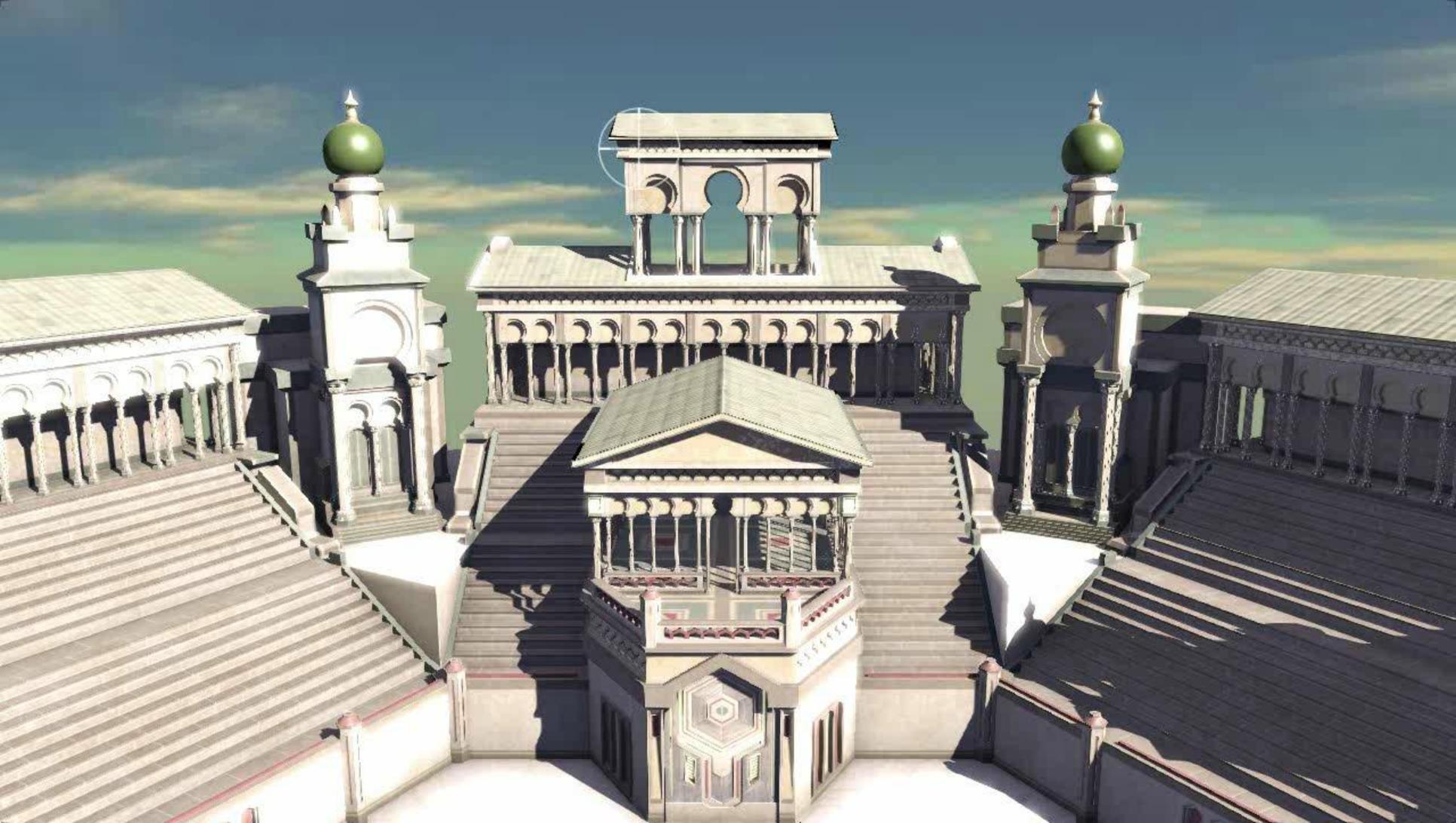


# Arena Destruction

(SG 2013 real time live)

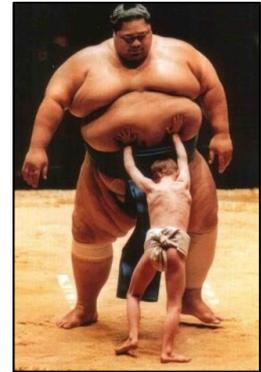
- 500k faces at start
- GPU1: rigid body simulation
- GPU2: smoke, rendering
- CPU: dynamic fracturing





# Deformable Objects

- 1d: Ropes, hair
- 2d: Cloth, clothing
- 3d: Fat guys, tires



# Existing Methods

- Force based
- Mass-Spring Systems / FEM
- Explicit integration **unstable**
- Implicit integration
  - Expensive
  - Large time steps for real time simulation needed
  - **Numerical damping**

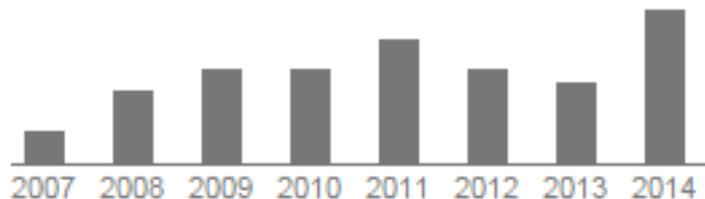
# Position Based Dynamics

[Müller et al., 2006]



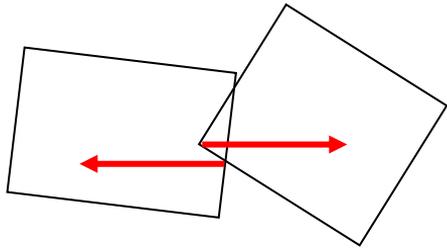
# Position Based Dynamics

[Müller et al., 2006]

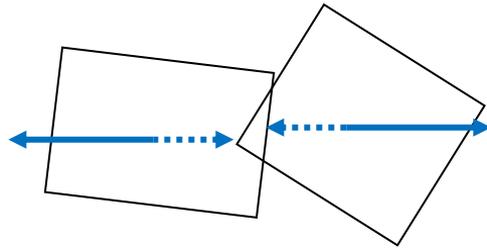


[google scholar]

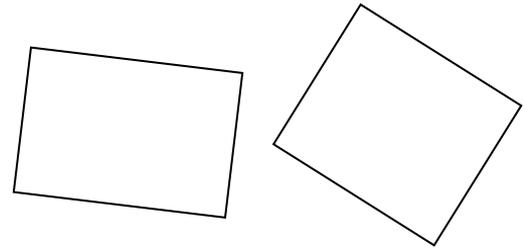
# Force Based Update



penetration  
causes forces



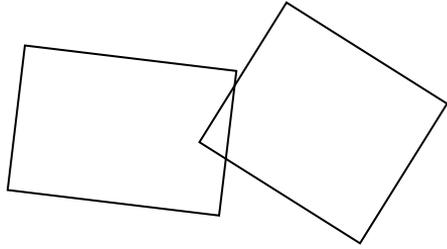
forces  
change velocities



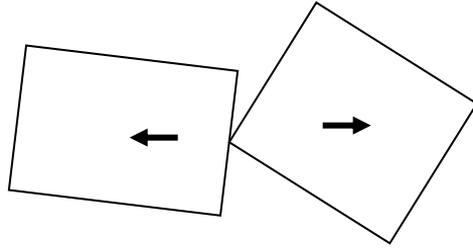
velocities  
change positions

- Reaction lag
- Small spring stiffness → squashy system
- Large spring stiffness → stiff system, overshooting

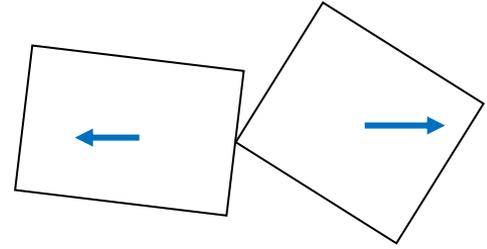
# Position Based Update



penetration  
detection only



move objects so that  
they do not penetrate



update velocities!

- Controlled position change
- Only as much as needed → no overshooting
- Velocity update needed to get 2<sup>nd</sup> order system!

# Position Based Integration

init  $\mathbf{x}_0, \mathbf{v}_0$

**loop**

$\mathbf{p} \leftarrow \mathbf{x}_n + \Delta t \cdot \mathbf{v}_n$

$\mathbf{x}_{n+1} \leftarrow \text{modify } \mathbf{p}$

$\mathbf{u} \leftarrow (\mathbf{x}_{n+1} - \mathbf{x}_n) / \Delta t$

$\mathbf{v}_{n+1} \leftarrow \text{modify } \mathbf{u}$

**end loop**

$\mathbf{x}_n, \mathbf{v}_n, \mathbf{p}, \mathbf{u} \in \mathbb{R}^{3N}$

prediction

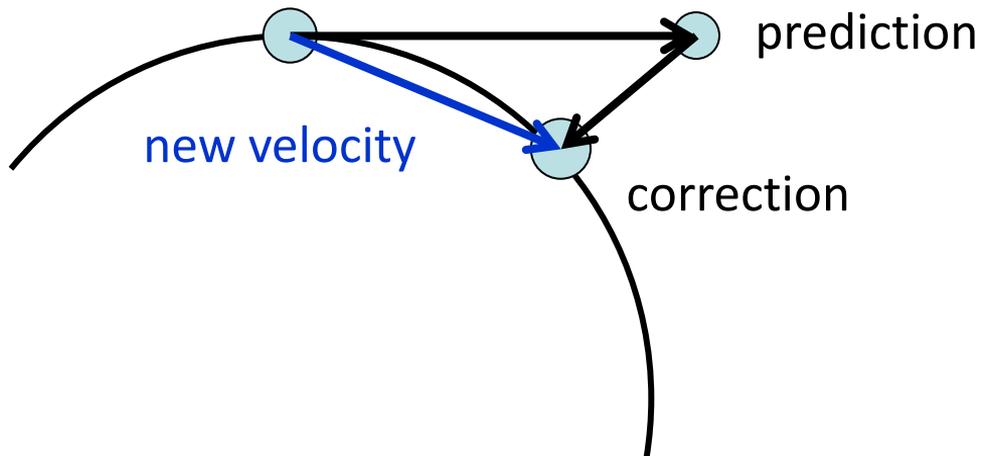
position correction

velocity update

velocity correction

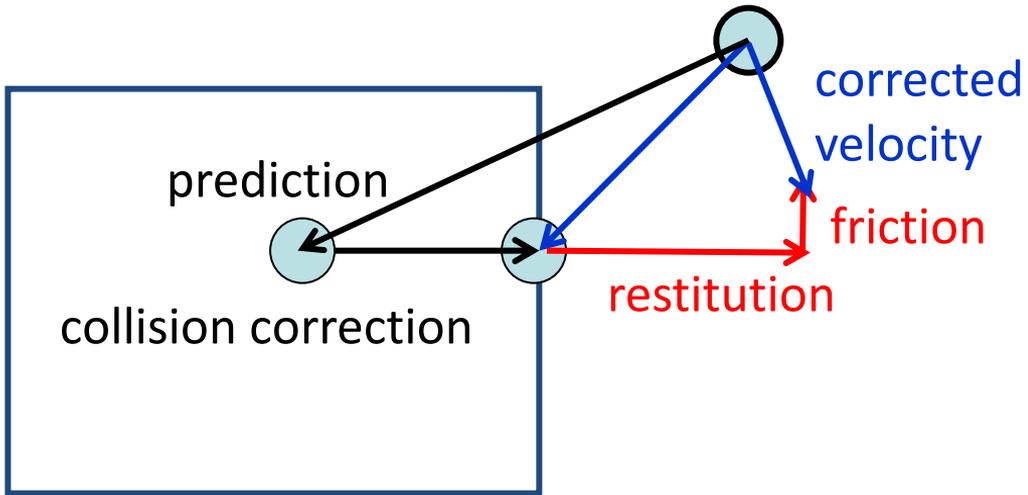
# Position Correction

- Example: Particle on circle

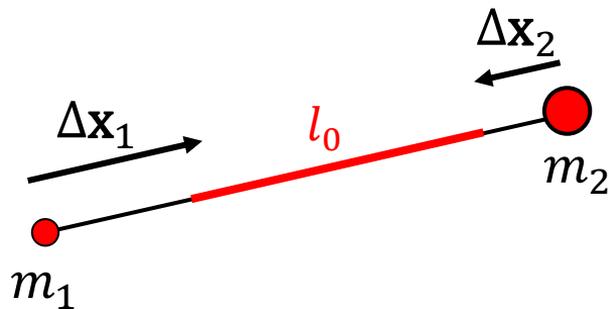


# Velocity Correction

- External forces:  $\mathbf{v}_{n+1} = \mathbf{u} + \Delta t \frac{\mathbf{f}}{m}$
- Internal damping
- Friction
- Restitution



# Distance Constraint



$$\Delta \mathbf{x}_1 = -\frac{w_1}{w_1 + w_2} (|\mathbf{x}_1 - \mathbf{x}_2| - l_0) \frac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$$\Delta \mathbf{x}_2 = +\frac{w_2}{w_1 + w_2} (|\mathbf{x}_1 - \mathbf{x}_2| - l_0) \frac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$$w_i = \frac{1}{m_i}$$

- Conservation of momentum
- Stiffness: scale corrections by  $k \in [0,1]$ 
  - Easy to tune
  - Effect dependent on **time step size** and **iteration count**
  - Often constant in games

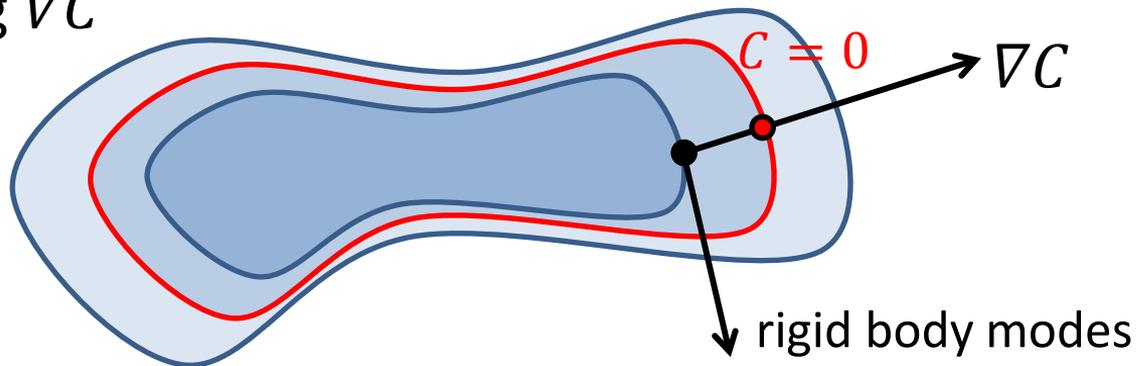
# General Internal Constraint

- Define constraint via scalar function:

$$C_{dist}(\mathbf{x}_1, \mathbf{x}_2) = |\mathbf{x}_1 - \mathbf{x}_2| - l_0$$

$$C_{volume}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) = [(\mathbf{x}_2 - \mathbf{x}_1) \times (\mathbf{x}_3 - \mathbf{x}_1)] \cdot (\mathbf{x}_4 - \mathbf{x}_1) - 6v_0$$

- Find configuration for which  $C = 0$
- Search along  $\nabla C$



# Constraint Projection

$$C(\mathbf{x} + \Delta\mathbf{x}) = 0$$

- Linearization (equal for distance constraint)

$$C(\mathbf{x} + \Delta\mathbf{x}) \approx C(\mathbf{x}) + \nabla C(\mathbf{x})^T \Delta\mathbf{x} = 0$$

- Correction vectors

$$\Delta\mathbf{x} = \lambda \nabla C(\mathbf{x})$$

$$\lambda = -\frac{C(\mathbf{x})}{\nabla C(\mathbf{x})^T \nabla C(\mathbf{x})}$$

$$\Delta\mathbf{x} = \lambda \mathbf{M}^{-1} \nabla C(\mathbf{x})$$

$$\lambda = -\frac{C(\mathbf{x})}{\nabla C(\mathbf{x})^T \mathbf{M}^{-1} \nabla C(\mathbf{x})}$$

$$\mathbf{M} = \text{diag}(m_1, m_2, \dots, m_n)$$

# Constraint Solver

- Gauss-Seidel
  - Iterate through all constraints and apply projection
  - Perform multiple iterations
  - Simple to implement
  - Atomic operations required for parallelization
- Modified Jacobi
  - Process all constraints in parallel
  - Accumulate corrections
  - After each iteration, average corrections [Bridson et al., 2002]
- Both known for slow convergence

# Global Solver

[Goldenthal et al., 2007]

- Constraint vector

$$C(\mathbf{x}) = \begin{bmatrix} C_1(\mathbf{x}) \\ \dots \\ C_M(\mathbf{x}) \end{bmatrix} \quad \nabla C(\mathbf{x}) = \begin{bmatrix} \nabla C_1(\mathbf{x})^T \\ \dots \\ \nabla C_M(\mathbf{x})^T \end{bmatrix} \quad \lambda = \begin{bmatrix} \lambda_1 \\ \dots \\ \lambda_M \end{bmatrix}$$

$$\Delta \mathbf{x} = M^{-1} \nabla C(\mathbf{x}) \lambda$$

$$\lambda = - \frac{C(\mathbf{x})}{\nabla C(\mathbf{x})^T M^{-1} \nabla C(\mathbf{x})}$$

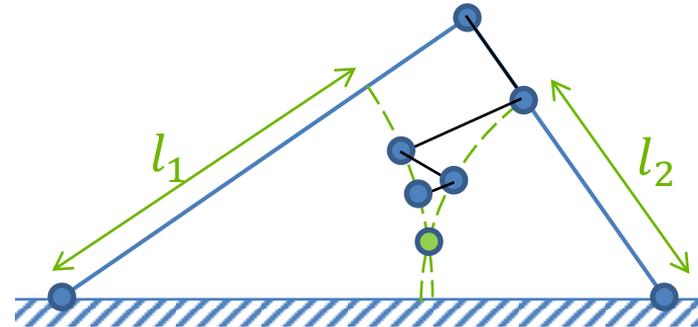
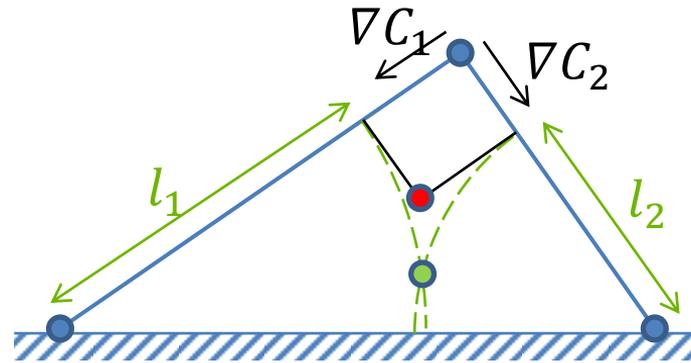


$$\Delta \mathbf{x} = M^{-1} \nabla C(\mathbf{x})^T \lambda$$

$$[\nabla C(\mathbf{x}) M^{-1} \nabla C(\mathbf{x})^T] \lambda = -C(\mathbf{x})$$

# Global vs. Gauss-Seidel

- Gradients fixed
- Linear solution  $\neq$  true solution
- Multiple Newton steps necessary
- Current gradients at each constraint projection
- Solver converges to the true solution



# Other Speedup Tricks

- Use as smoother in a multi-grid method
- Long range distance constraints (LRA)
- Shape matching
- Hierarchy of meshes

# Amazing Gauss-Seidel!

- Can handle unilateral (inequality) constraints (LCPs, QPs)!
  - Fluids: **separating boundary conditions** [Chentanez et al., 2012]
  - Rigid bodies: LCP solver [Tonge et al., 2012]
  - Deformable objects: Long range attachments [Kim et al., 2012]
- Works on **non-linear problem** directly
- Handles under and over-constrained problems
- GS + PBD: garbage in, simulation out (almost 😊)
- **Fine grained interleaved solver** trivial
- Easy to implement and parallelize

# Analysis of PBD

# Correction = Acceleration

- Predicted position

$$\mathbf{p} = \mathbf{x}_n + \Delta t \mathbf{v}_n = \mathbf{x}_n + \Delta t \frac{(\mathbf{x}_n - \mathbf{x}_{n-1})}{\Delta t} = 2\mathbf{x}_n - \mathbf{x}_{n-1}$$

- Projection

$$\mathbf{x}_{n+1} = \mathbf{p} + \Delta \mathbf{x}$$

$$\Delta \mathbf{x} = \mathbf{x}_{n+1} - 2\mathbf{x}_n + \mathbf{x}_{n-1}$$

# Implicit Euler

$$M \frac{\mathbf{x}_{n+1} - 2\mathbf{x}_n + \mathbf{x}_{n-1}}{\Delta t^2} = \mathbf{f}(\mathbf{x}_{n+1})$$

$$M\Delta\mathbf{x} = \Delta t^2 \mathbf{f}(\mathbf{x}_{n+1})$$

Formulation as an optimization problem for  $\Delta\mathbf{x}$ :

$$\min \left( \underbrace{\frac{1}{2} \Delta\mathbf{x}^T M \Delta\mathbf{x}}_{\text{inertia term}} + \underbrace{\Delta t^2 E(\mathbf{x}_{n+1})}_{\text{energy term}} \right)$$

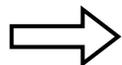
# Stiffness $\rightarrow$ Infinity

$$\min \left( \frac{1}{2} \Delta \mathbf{x}^T \mathbf{M} \Delta \mathbf{x} + \Delta t^2 \frac{1}{2} k C^2(\mathbf{x}_{n+1}) \right) \quad // E(\mathbf{x}) = \frac{1}{2} k C^2(\mathbf{x})$$

Now let  $k \rightarrow \infty$

$$\min \left( \frac{1}{2} \Delta \mathbf{x}^T \mathbf{M} \Delta \mathbf{x} \right) \text{ subject to } C(\mathbf{x}_{n+1}) = 0$$

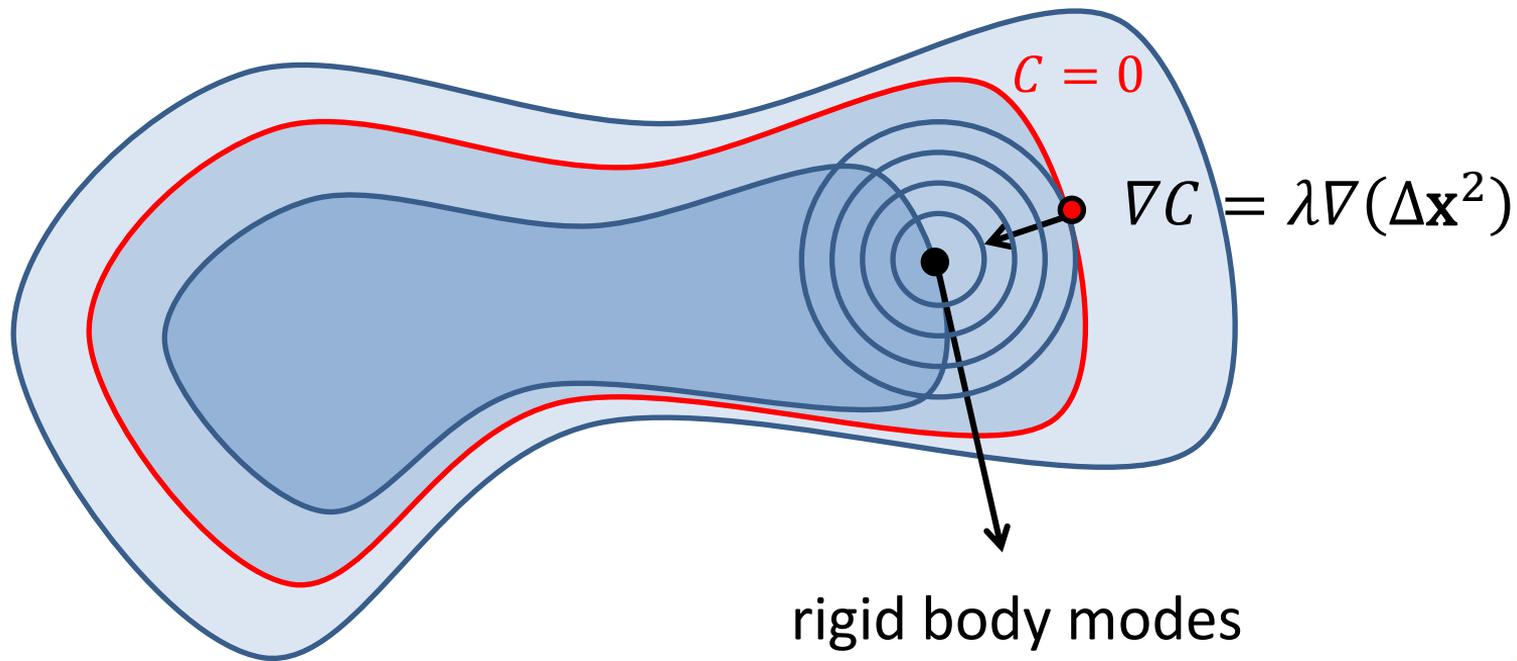
- $C(\mathbf{x}_{n+1}) = 0$
- $\mathbf{M} \Delta \mathbf{x} = \lambda \nabla C(\mathbf{x}_{n+1})$



$$\Delta \mathbf{x} = \lambda \mathbf{M}^{-1} \nabla C(\mathbf{x}_{n+1})$$

PBD

# Two Interpretations



# Constraint Solver

- PBD solves a non-linear optimization problem

$$\min \left( \frac{1}{2} \Delta \mathbf{x}^T \mathbf{M} \Delta \mathbf{x} \right) \text{ subject to } C_i(\mathbf{x}_{n+1}) = 0, \quad i \in [1, \dots, m]$$

by solving a **sequence of QPs**:

$$\min \left( \frac{1}{2} \Delta \mathbf{x}^T \mathbf{M} \Delta \mathbf{x} \right) \text{ subject to } C_i(\mathbf{x}_{n+1}) = 0$$

# Clothing Demo

Nurien

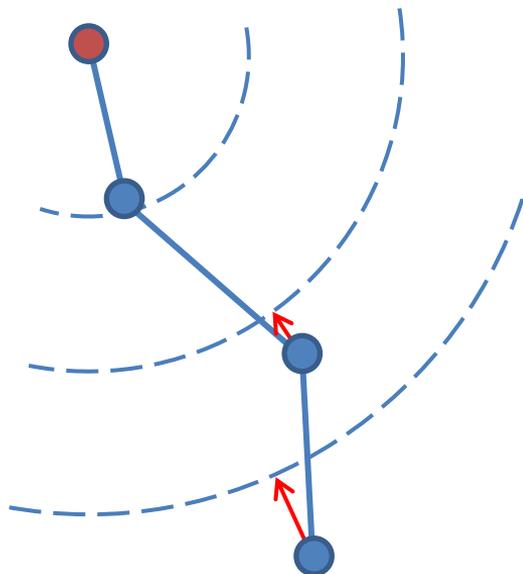


# Cloth

- Slow error propagation → stretchy cloth
- Low resolution: no detailed wrinkles
- Solutions
  - Use hierarchy of meshes (complicated)
  - Has been an open problem for us
  - Found an embarrassingly simple solution

# Long Range Attachments (LRA)

- Upper distance constraint to closest attachment point
- Unilateral: project only if distance too big



[Kim et al., 2012], 90k particles

850k particles, 100k hairs, 27.29 fps

Left Mouse - Blow

Right Mouse - Rotate Camera

3 Lights

8M/SAA

Supersampling

Shadow Opacity

Shadow Ambient

Per Hair Shadow Variation

Hair Opacity

Hair Base Thickness

Hair Tip Thickness

Hair Diffuse Color Scale

Hair Specular Color Scale

Hair Specularity

Fluffiness

Wind X  Wind Y  Wind Z

Blower Speed  Blower Radius

Animate (A)

Simulate (S)

Slim Windy (1)

Normal Windy (2)

Fluffy Windy (3)

Slim No Wind (4)

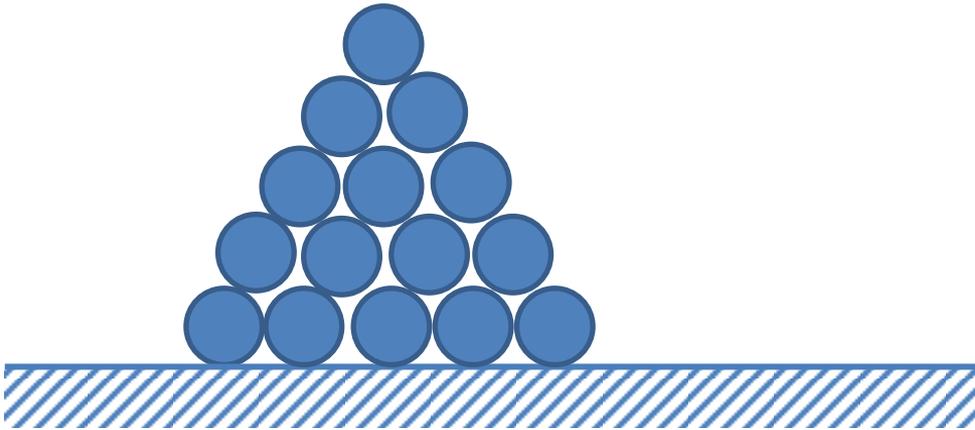
Normal No Wind (5)

Fluffy No Wind (6)

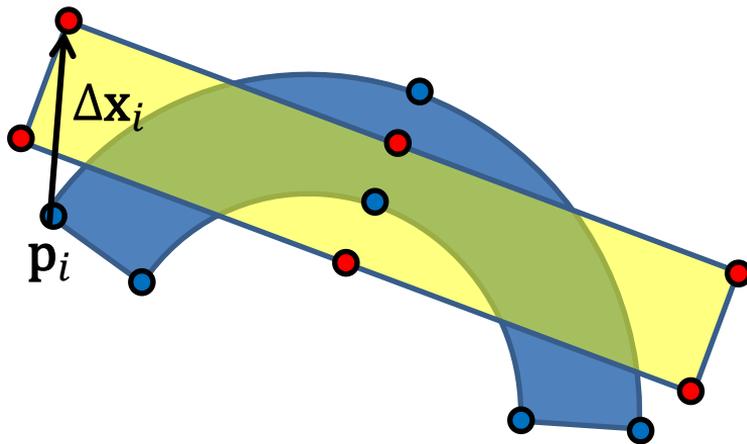


# Challenge

- Similar idea for compression?
- Long range distance constraint to the ground?



# Rigid Objects



- Optimally match un-deformed with deformed shape
- Only allow translation and rotation
- Global correction, no propagation needed
- No mesh needed!

# Position Based Fluids

[Macklin et al. 2013]

- Particle based
- Pair-wise lower distance constraints  
→ granular behavior
- Move particles in local neighborhood  
such that density = rest density
- Density constraint

$$C(\mathbf{x}_1, \dots, \mathbf{x}_n) = \rho_{SPH}(\mathbf{x}_1, \dots, \mathbf{x}_n) - \rho_0$$



# Mesh Independent Deformations



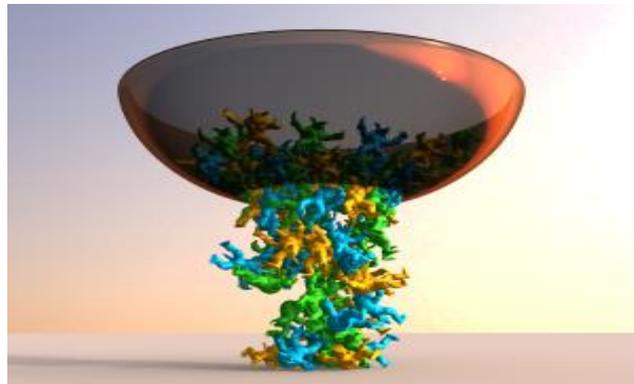
[Müller et al, 2014]

- For each triangle:

$$C(\mathbf{x}_1, \dots, \mathbf{x}_3) = \mathbf{G}_{ij}(\mathbf{x}_1, \dots, \mathbf{x}_3)$$

$$\mathbf{G} = \mathbf{F}^T \mathbf{F} - \mathbf{I}$$

# FEM



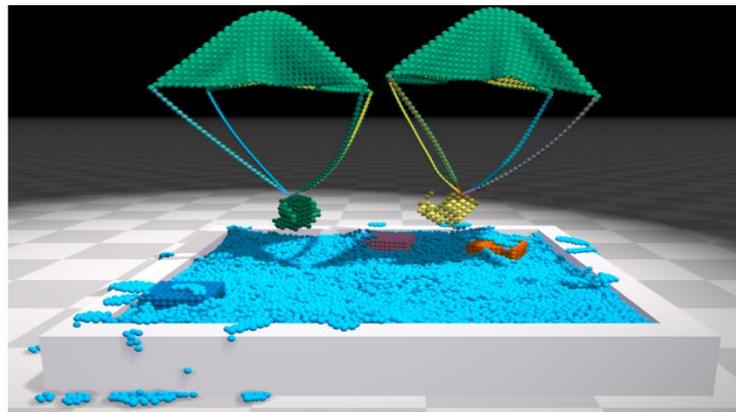
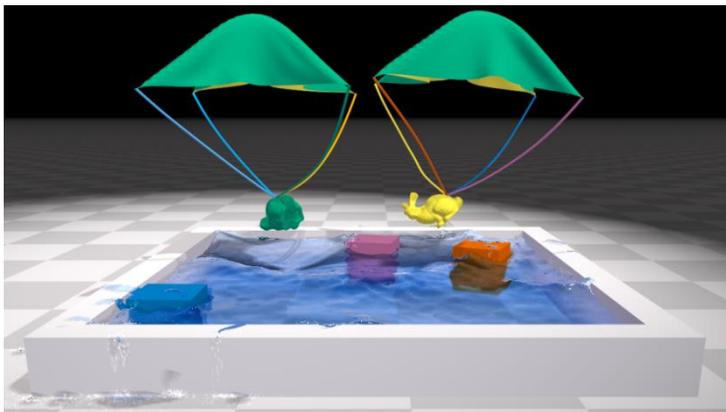
[Bender et al, 2014]

- For each tetrahedron:

$$C(\mathbf{x}_1, \dots, \mathbf{x}_4) = E_{FEM}(\mathbf{x}_1, \dots, \mathbf{x}_4)$$

# Unified Solver

[Macklin et al., 2014]



- Putting it all together
- Plus
  - Static friction
  - Stiff stacks via mass modifications
  - Two-way fluid – solid coupling



# Acknowledgements

- PhysX Research Group



Nuttapon  
Chentanez



Tae-Yong  
Kim



Miles  
Macklin

- PhysX Group



# Thanks!

# Questions?

Scene 8 : Arena (GRB)

General:

p: pause

o: single step

w,a,s,d,q,e: camera

f: toggle full screen mode

F1: toggle help

F4: change weapon: LMB

Shift+mouse: drag object

